LOCALLY RESONANT METAMATERIALS FOR UNDERWATER ACOUSTIC APPLICATIONS: COMPARISON OF AN ANALYTICAL MODEL WITH EXPERIMENTAL RESULTS

Christian Audoly

Naval Group Research, 199 Avenue Pierre-Gilles de Gennes, 83190 Ollioules, France
e-mail: christian.audoly@naval-group.com

Valentin Leroy

Laboratoire Matière et Systèmes Complexes, CNRS/Université de Paris, 75205 Paris Cedex 13, France
e-mail: valentin.leroy@univ-paris-diderot.fr

Acoustic materials and coatings represent a key technology for naval applications such as the reduction of underwater radiated noise and target strength of submerged structures or underwater vehicles. They can also be relevant for civilian applications, such as the reduction of noise emitted by industrial activity at sea for protection of marine life. Due to the limitations of classical micro-voided material technology, in particular at low frequency, concepts of acoustic metamaterials exhibiting locally resonant phenomena are of great interest. This paper focuses on acoustic panels formed by a 2D periodic or random distribution of voids in a viscoelastic layer such as polyurethane. Analytic models have been developed to predict their performance, such as the transmission coefficient as a function of frequency. The main objective of this paper is to compare these simple models to experimental data. It is shown that there is an overall good match between theory and experiment, including for random distributions.

Keywords: metamaterials, underwater acoustics

1. Introduction

For more than two decades now, acoustic metamaterials have been a hot research topic in many laboratories around the world, with a wide range of potential applications ranging from protection of buildings from seismic very low frequency vibration to devices for control of ultrasonic surface waves in electronic components, also including airborne auditory acoustics and underwater acoustics. The benefit of these tailored artificial materials, by comparison to natural or more classical materials used in industry, is the possibility to obtain unusual dynamic properties such as negative effective properties, high damping factor, or high transmission loss or absorption in sub-wavelength conditions. For the applications in underwater acoustics, the materials take generally the form of coatings, a few centimeters thick, to be installed on the hulls of submerged structures. There are two main functions of interest [1]:

- Decoupling (or low transmission) coatings, to reduce radiated sound when the structure is submitted to internal excitation,
- Anechoic coatings, to reduce sound reflected or backscattered by the structure excited by an incoming acoustic wave.

The frequencies of interest range typically from very low frequency up to several tens of kHz.

However, it should be noted that:

- A lot of literature and patents are available for applications in airborne acoustics, but much less in underwater acoustics. Also, the technological solutions working in air are not readily transposable to water due to the large difference in the acoustic properties of the surrounding medium.
- The literature is abundant on theoretical aspects, much less on experimental results.

Research on acoustic metamaterials for underwater applications is currently very active but pioneer developments have been carried out a long time ago. In particular, the name of the "Alberich" material concept comes from the historical rubber coating installed on a German submarine during WWII (left of Fig. 1), which was designed with lattices of resonant cavities of appropriate diameter corresponding to frequencies to be absorbed. Although recent research considers advanced metamaterial with complex internal structure, the basic concept of Alberich material, idealized in the form of a periodic arrangement of spherical cavities embedded in an elastomeric slab (Fig. 1) is still worth of interest as it allows designing decoupling or anechoic coatings with significant performance. As a matter of fact, the performance appears around a resonant frequency depending of the dynamic properties of the elastic slab, the radius *a* of the voided inclusions, the spacing *d* in the lattice, and the thickness *e* of the layer. If necessary, several layers can be combined to increase the frequency band of efficiency.

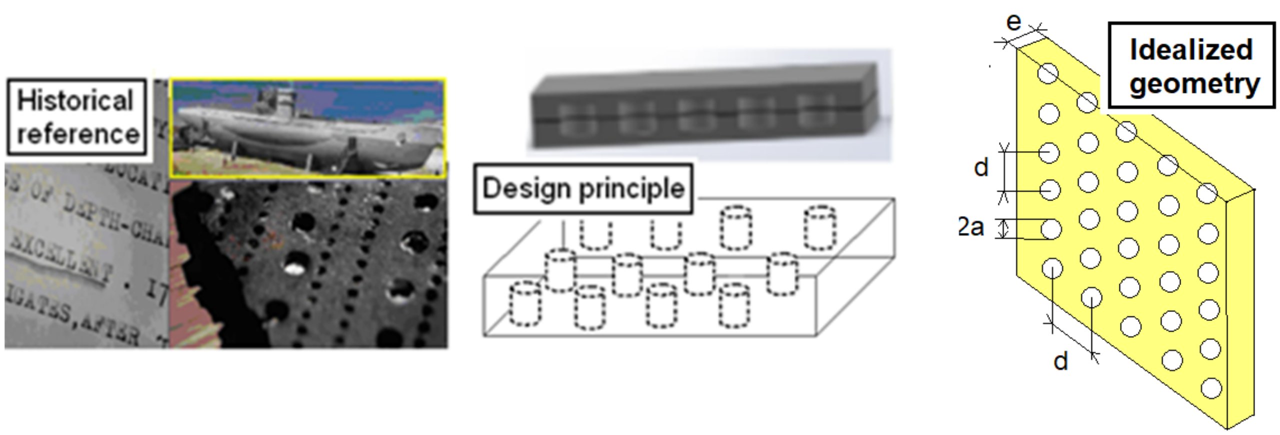

Figure 1: Principle of the Alberich-type acoustic coating, and geometrical parameters used in this study

For design purposes it is important to have at one's disposal reliable tools for the prediction of acoustic performance of these structures, in terms of reflection and transmission coefficients in water. In early research, Gaunaurd *et al*. [2] showed that the acoustic behavior of the material was due to a monopole resonance related to the diameter of the inclusion and to the dynamic shear modulus of the matrix. This methodology, as well as other theories such as Waterman's [3], can be used to predict effective acoustic properties of composites composed of a random distribution of resonant inclusions in a matrix. At the beginning of the 90's, an important milestone was the development of a finite element method capable of predicting the acoustic performance of materials with arbitrary periodic structure [4]. Since then, different commercial softwares have implemented a similar functionality. In that context, the purpose of the experiments reported in [5] was obtaining data for validation of the numerical technique based on finite elements (new at that time), and the retrieve of effective properties to be compared to theory. Reference [5] also includes results on pseudo-random configuration, to be compared with the effective medium theories mentioned above.

More recently, Leroy *et al.* developed an analytical model for the modeling of acoustic properties of arrays of bubbles [6], which proved to be relevant for Alberich coatings [7]. Indeed, a voided inclusion in a soft matrix has an acoustic response that is similar to that of a bubble, provided that the wavelength remains much larger than the inclusion. The model was successfully compared to experimental results and simulations, as long as the distance between the inclusions remained large enough: in practice for $d/a>5$. For higher concentrations, it is suspected that the monopolar approximation for the response of the cavities is not appropriate [8].

Considering these recent developments, the objective of this paper is to revisit the experimental data presented in references [4] and [5] by confronting with prediction. Indeed, there is still little data available for well-controlled experiments in water, in particular in the low frequency domain, requiring large size samples (typically one square meter and a few centimeters thick). On the other hand, the theory by Leroy allows obtaining quick results using only analytical formulae, providing also physical interpretation. The paper will describe first the samples and the acoustic measurement technique in a water tank. Then the theoretical model will be presented, applied to the different samples with corresponding input data, and compared to experiment. The results will be discussed in the last section.

2. Description of the test samples and measurement technique

2.1 Description of test samples

During previous studies, different test panels have been designed and built on-purpose for laboratory testing. The main objective was not application-oriented industrial development and optimization, but the knowledge of physical phenomena, including comparison with effective medium theories [5] and validation of numerical methods [4].

Seven test panels, whose dimensional and physical parameters are summarized in Table 1, are considered. Test panels N°1-6 have already been described in [5] and test panels 6 and 7 in [4]. In order to comply as much as possible with theory using analytical models, test panels N°1-5 were designed with voided inclusions of spherical shape. Also, test panels N°1-2 had a pseudo-random distribution of inclusions in three layers (n=3), whereas for panels N°3-7 the distribution was periodic in one layer (n=1). Regarding the choice of the matrix, two elastomers were selected, one of polyurethane type (a relatively hard elastomer), one of silicone type (a soft elastomer). Contrary to rubber, that requires a vulcanization process with adequate molds and facilities, the selected elastomers can be casted in laboratory at room temperature, passing from liquid to solid state through a polymerization process specified by the provider. The practical manufacture of the test panels was rather difficult, in particular for the insertion of the voided inclusions of spherical shape at selected locations within the matrix. To achieve the final geometry, several layers were casted, then assembled together using the same elastomer. Special care has been taken to avoid trapping air bubbles at the interface between the sub-layers. Figure 2 presents two examples of schematics of these test panels, and additional information can be found in ref. [4] and [5].

Table 1: Parameters of the 7 test panels. Geometrical parameters are: the number of layers n, the radius of the cavities *a*, the distance between cavities *d*, and the thickness of the panel *e*. The properties of the matrix are given by five parameters: its mass density ρ, its complex shear modulus $G(1+i\eta_G)$ and its complex longitudinal velocity $c_L(1+itg\delta_L)$.

Panels		Geometry				Matrix				
Number	Type	n	*a* (mm)	*d* (mm)	*e* (mm)	ρ (kg/m^3)	*G* (MPa)	η_G	c_L (m/s)	$tg\delta_L$
1	Random	3	10	95(*)	70	1100	32	0.42	1540	0.05
2	Random	3	10	42(*)	70	1100	32	0.42	1540	0.05
3	Periodic	1	10	100	50	1000	1	0.2	1000	0.05
4	Periodic	1	10	100	50	1100	32	0.42	1540	0.05
5	Periodic	1	10	50	50	1100	32	0.42	1540	0.05
6	Periodic	1	9.4(**)	50	40	1100	36	0.42	1540	0.05
7	Periodic	1	9.4(**)	50	40	1000	0.6	0.15	1100	0.03

Notes:

(*) For test panels 1 and 2, the distribution is pseudo-random, so the definition of *d* as on figure 1 cannot be used. Here, *d* is defined as the average distance between one inclusion to the nearest one.

(**) For test panels 6 and 7, the actual shape of the inclusions is a short cylinder instead of a sphere. In that case, the value of *a* is the radius of the sphere corresponding to the same volume as the cylinder.

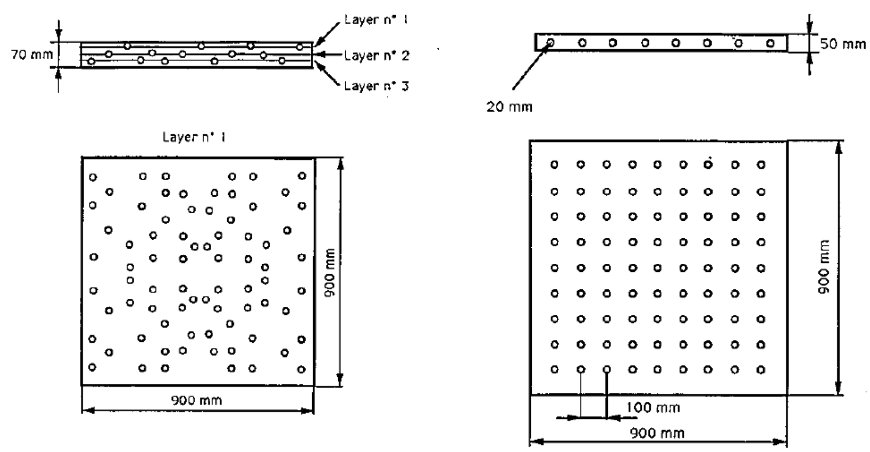

Figure 2: Internal structure of test panels N°1 (left), and N°3 (right), from [5].

For the modelling, the knowledge of the elastic dynamic properties of the elastomeric matrix is needed. The notations for the characteristics of the matrix in Table 1 are the same as those used in [5]. Considering that, in a formulation in the frequency domain, the physical quantities such as acoustic pressure are represented by complex numbers, the dynamic moduli or the corresponding wavenumbers are also complex quantities. Here we have selected the dynamic shear modulus and the longitudinal wave speed. It is important to note that the dynamic moduli can vary with frequency and temperature, so that special procedures and devices must be used for their characterization [9]. Despite the fact that the variation of moduli with frequency can be significant, it may be sufficient in some cases to consider a constant value. The data in the right-hand side of table 1 are in fact typical values at 5 kHz. We can see in table 1 that the shear modulus for the polyurethane (panels N°1-2 and N°4-6) is much higher than for the silicone (panels N°3 and N°7). For test panel N°6, the same polyurethane matrix is used, but the shear modulus is slightly greater due to the lower temperature during acoustic measurement.

2.2 Measurement technique

The frequencies of interest being in the kHz range, the size of the test panels is large, and as a consequence a water tank facility several meters in length, width and depth, is needed. Here, most of measurements were done at the former DCN Toulon/GERDSM laboratory, and additional ones at IEMN/ISEN, Lille. The technique used, illustrated in Fig. 3 and described in ref. [9] and [10] is the "insertion technique" in the frequency domain. A piezoelectric transducer installed in the water tank generates an underwater acoustic wave at a given frequency in a given time window. The sound is received at some distance, a few meters, using a hydrophone. The first step is determining the transfer function between the hydrophone and the emitter, in the absence of the test panel, by varying the frequency. The second step consists in measuring again the transfer function, placing the test panel either in front of the hydrophone (for measurement of the transmission coefficient), or behind the hydrophone (for measurement of the combination of the incident and reflected wave). In both cases, the distance between the test panel and the hydrophone is a few centimeters. The physical quantity of interest is given by the ratio between the transfer functions from the two steps. It should be noted that the obtained quantities are complex-valued, allowing determining not only the amplitude (or level in dB) but also the phase.

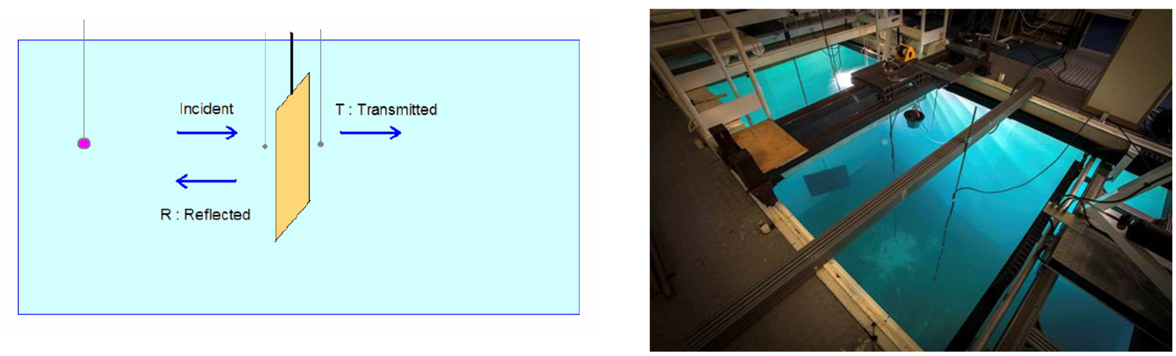

Figure 3: Principle of the acoustic measurement of a test panel and example of water tank facility

3. Modelling and comparison to experiment

The model is based on the reflection (r) and transmission (t) coefficients for an array of spherical cavities in a matrix, at angular frequency ω [5]:

$$r = \frac{iKa}{\left(\frac{\omega_0}{\omega}\right)^2 - I - i(\delta + Ka)} \tag{1}$$

$$t = 1 + r \tag{2}$$

where a is the radius of the cavities, d the distance between the cavities, $K = 2\pi/kd^2$, $\omega_0^2 = 4G'/\rho a^2$, $I = 1 - Ka\sin(kd/\sqrt{\pi})$ and $\delta = 4G''/\rho a^2\omega^2$. Parameters of the matrix are: $k = \omega/c$, the wavenumber of longitudinal waves (c is the celerity), G', G'' the real and imaginary parts of the shear modulus, and ρ the density.

For going from the response of a single layer to a multi-layer, we use an iterative process. If we add to a structure with transmission coefficient T_n and reflection coefficient R_n, a new layer of thickness e_n, the total coefficients for the new structure will be:

$$T_{n+1} = \frac{t_{n+1,n} T_n e^{ik_n e_n}}{1 - r_{n,n+1} R_n e^{2ik_n e_n}}$$

$$R_{n+1} = r_{n+1,n} + \frac{t_{n+1,n} t_{n,n+1} R_n e^{2ik_n e_n}}{1 - r_{n,n+1} R_n e^{2ik_n e_n}}$$

where $t_{n,n+1}$ is the transmission coefficient when going from medium n to medium n+1, and $r_{n,n+1}$the reflection coefficient. When the interface is between two media of different impedances, $t_{n,n+1} = 2Z_{n+1}/(Z_n + Z_{n+1})$ and $r_{n,n+1}$=$(Z_{n+1} - Z_n)/(Z_{n+1} + Z_n)$. And when the interface is a layer of cavities, we use equations (1) and (2). This procedure is useful for computing the response of the samples with 3 layers, but also for the single layers because it allows us to take into account the propagation in the matrix (with attenuation) and the multiple reflections that occur.

In Figures 4 to 6, the symbols represent the experimental transmission coefficient taken from [4-5], (amplitude in dB and phase in degrees referred to the incident plane wave), and the solid curves represent the corresponding prediction using the analytical model. The black solid line represents the transmission coefficient of the viscoelastic layer without any inclusion.

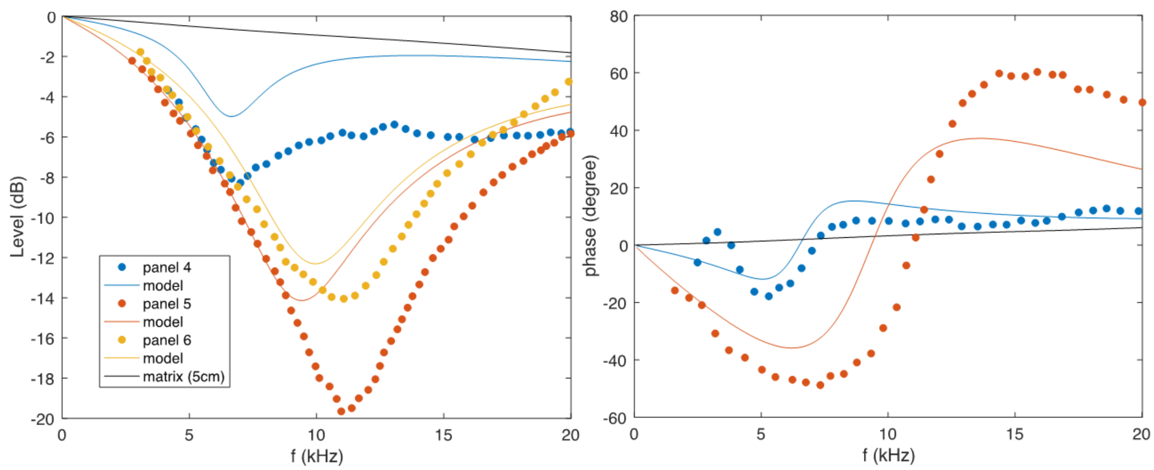

Figure 4: Results for tests panels n°4-6, with periodic distribution of voids and polyurethane matrix

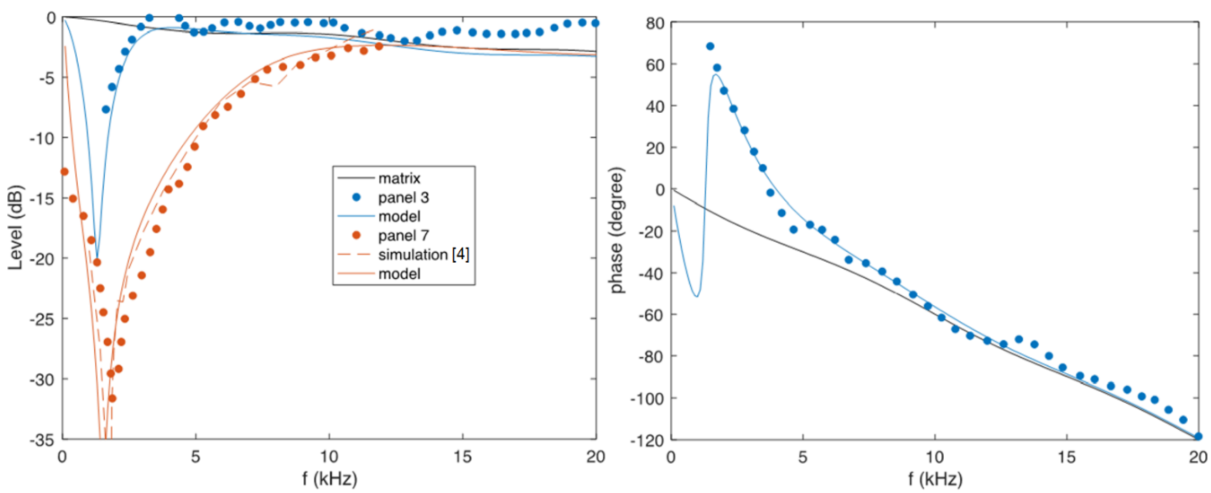

Figure 5: Results for tests panels n°3 and 7, with periodic distribution of voids and silicone matrix

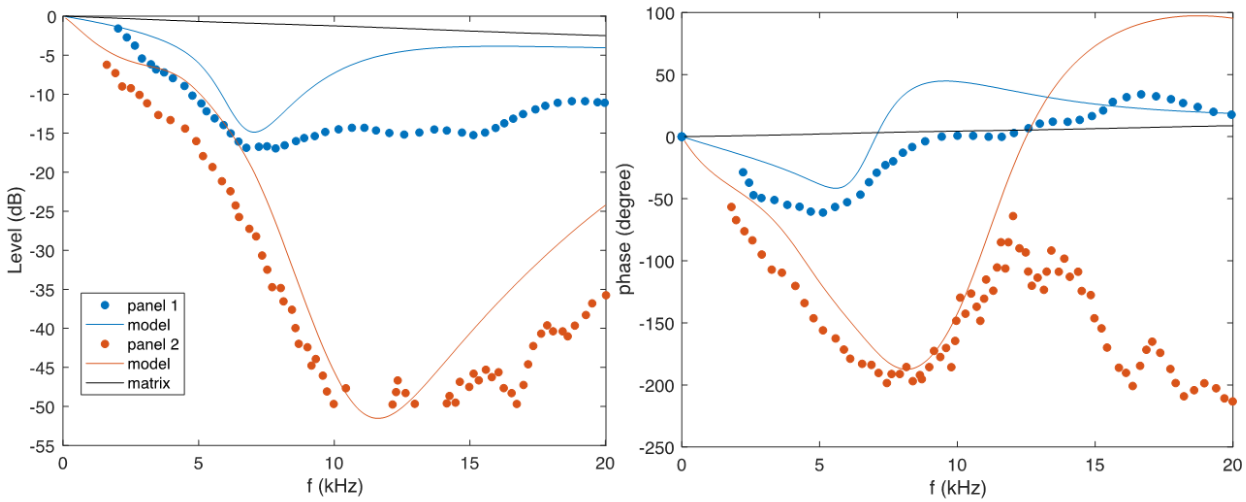

Figure 6: Results for tests panels N°1 and 2, with random distribution of voids and polyurethane matrix

4. Discussion

In the case of the test panels with periodic distribution of voids and the silicone matrix, Fig. 5, the agreement between prediction and experiment is quite good. Furthermore, referring to [4] the analytical model used here provides results as accurate as the finite element method for panel N°7. With panel N°3 the resonance frequency is very low, approximately 1 kHz, with a narrow band of efficiency. When increasing the volume ratio of inclusions, by reducing the grating spacing *d*, we observe an increase of the resonance frequency and of the bandwidth of low transmission. At resonance the transmission is as low as -30 dB.

We consider now the case of the test panels with periodic distribution of voids and the polyurethane matrix, Fig. 4. Of course, we find that the resonant frequency is higher here, because the dynamic stiffness of the polyurethane matrix is much greater than the silicone. For test panel N°5, there is a good match between prediction and experiment, and with the results using the finite element method [4], with however an underestimation of the resonance frequency by 10%. This may be due to the fact that the variation of the dynamic moduli of the polyurethane with frequency is not taken into account here. In particular, the shear modulus is expected to be greater at 10 kHz than at 5 kHz, the reference value taken here. For test panel N°4, the resonance frequency, near 5 kHz, is well predicted, but the experimental transmission coefficient is lower than predicted. However, the variation of the phase with frequency is well predicted. The lower transmission coefficient is also observed with test panel N°6 above the resonant frequency.

Although the theory presented in section 3 applies to periodic distributions, not random ones, the comparison with experiment was also done for test panels N°1 and 2, Fig. 6, using an average distance between one inclusion and its nearest. Surprisingly, the model matches reasonably well the experimental results. For test panel N°1 (the lowest concentration of voids) the predicted transmission coefficient is higher than experiment, in a similar way as for the periodic configurations. For test panel N°2 (with high concentration of voids), we observe a bias on the level, which can probably be explained by the variation of the shear modulus with frequency, not taken into account here. There is also a discrepancy on the phase above the resonant frequency. A possible interpretation is the uncertainty in obtaining experimentally the phase when the transmission coefficient is very low, in particular at high frequency.

In conclusion, the analytical model presented here in section 3 was found to provide good results to predict the acoustic performance of low frequency Alberich materials for underwater applications. Although it cannot replace the finite element method for metamaterial with complex and arbitrary internal structure, it provides relevant results for the type of metamaterials considered here, with very little computational effort, providing also a direct interpretation of the observed phenomena. In particular, we have shown the effect on the resonant frequency of the dynamic properties of the matrix and of the geometry of the grating (diameter of the inclusions and their spacing). Also, in the case of voided inclusions treated here, we also confirmed the monopole behaviour of the resonance, as different shapes of inclusion (sphere or short cylinder) lead to similar results.

REFERENCES

1 Audoly, C., Acoustic metamaterials and underwater acoustics applications. *In "Fundamentals and Applications of Acoustic Metamaterials: From Seismic to Radio Frequency", Volume 1.* Ed. Wiley, Book Editor(s): Vicente Romero-García and Anne-Christine Hladky-Hennion. First published: 06 August 2019. https://doi.org/10.1002/9781119649182.ch9

2 Gaunaurd, G. C., Überall, H, *Resonance theory of the effective properties of perforated solids.* J. Acoust. Soc. Am, Vol. 73, p. 372 (1983)

3 Waterman, P.C., Truell, R., *Multiple scattering of waves*. J. Math. Phys., 2, 512 (1961)

4 Hladky-Hennion, A. C., Decarpigny, J. N., *Analysis of the scattering of a plane acoustic wave by a doubly periodic structure using the finite element method: Application to Alberich anechoic coatings.* J. Acoust. Soc. Am, Vol. 90, p. 3356 (1991)

5 Audoly, C., *Acoustic analysis of panels made with viscoelastic materials containing resonant cavities*. Acta Acustica, Vol. 2, N°5 (1994)

6 Leroy, V., Strybulevych, A., Scanlon, M.G., and Page, J.H., *Transmission of ultrasound through a single layer of bubbles*, Eur. Phys. J. E **29**, 123 (2009)

7 Leroy, V., Strybulevych, A., Lanoy, M., Lemoult, F., Tourin, A., and Page, J.H., *Super-Absorption of Acoustic Waves with Bubble Meta-Screens*, Phys. Rev. B **91** (2015)

8 Leroy, V., Chastrette, N., Thieury, M., Lombard, O., and Tourin, A., *Acoustics of Bubble Arrays : Role Played by the Dipole Response of Bubbles*, Fluids **3** (2018)

9 Audoly, C., *Characterization of underwater passive materials- Introduction.* Journal of Wave-Material Interaction, Vol. 5-6, N°1-2, p. 1-18 (1991)

10 Audoly, C., Giangreco, C., *Improvement of the measurement of the transmission coefficient of panels at normal incidence using surface receivers.* J. Acoustique Vol.3 p.369-379 (1990)